Abstract

Wall functions are widely used in commercial CFD software and offer significant savings in computational expense compared to low-Reynolds-number formulations. However, existing wall functions are based on assumed near-wall profiles of velocity, turbulence parameters and temperature which are inapplicable in complex, non-equilibrium flows. A new wall function is developed in this thesis which, instead of assuming profiles of the dependent variables, determines these quantities by solving boundary-layer-type transport equations across a locally-defined subgrid.

The new wall function, called UMIST-*N*, is applied to three test cases: an axisymmetric impinging jet, a spinning "free" disc and a three-dimensional simplified car body. The impinging jet flow ($H/D = 4$; $Re = 70,000$) is studied using linear and non-linear $k - \varepsilon$ models with the UMIST-*N* wall function, four "standard" log-law-based wall functions and full low-*Re* treatments. It is demonstrated that heat transfer predictions with the UMIST-*N* wall function are in excellent agreement with low-*Re* model results, in contrast to standard log-law-based wall functions. The new wall function also shows less sensitivity to the size of the near-wall cell than standard wall functions.

Spinning-disc calculations are carried out at rotational Reynolds numbers up to $Re_\phi = 3.3 \times 10^6$ using a similar array of turbulence models and wall treatments. The UMIST-*N* wall function and low-*Re* model results are again in excellent agreement, in contrast to standard wall functions which are unable to predict correctly the radial velocity profile. The location of the predicted transition point from laminar to turbulent flow on the spinning-disc shows some slight sensitivity to the near-wall grid arrangement with the UMIST-*N* wall function, although the results are close to those obtained with the low-*Re* models.

Simulation of the simplified "Ahmed" body flow demonstrates that the UMIST-*N* wall function can be applied to complex geometry using a non-orthogonal multiblock grid. Flow predictions over the 25° rear slant of the car using UMIST-*N* with linear $k - \varepsilon$ model are shown to be similar to those obtained using a log-law-based wall function.

In the three test-cases considered, computing times with the new wall function are up to twice as high as for standard wall functions, but they are still an order-of-magnitude less than low-Reynolds-number calculations.

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